LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2025**

UMT 4402 – MATHEMATICS FOR PHYSICS - II

Date: 06-05-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Answer the following**
a)	Define Fourier series.
b)	Write the two forms of Leibnitz's equations of first order differential equation.
c)	What is the particular integral of $(D-2)^2y = 8\,sin2x$?
d)	Find the Laplace transform of $cos^2 2t$.
e)	Define vector point function.
2.	**Fill in the blanks**
a)	If $f(x)$ is an odd function, then the Fourier coefficient a_n is _______.
b)	The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^6 - 4\left(\frac{d^2y}{dx^2}\right)^8 + \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2 + y = 0$ are ____ and _____.
c)	The complementary function of $(D^2+4D+4)y = 4x+2$ is _______.
d)	The Laplace inverse transform of $\frac{s-1}{(s-1)^2+9}$ is ________.
e)	The total work done by the force $\boldsymbol{F}$ during the displacement from A to B equals to the line integral _________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**MCQ**
a)	Which of the following is not in Dirichlet's condition? (i) $f(x)$ is periodic, single valued and finite (ii) $f(x)$ has a finite number of discontinuities in any one period (iii) $f(x)$ has an infinite number of maxima and minima (iv) $f(x)$ has at the most a finite number of maxima and minima
b)	The necessary and sufficient condition for a differential equation $Mdx + Ndy = 0$ to be exact is (i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (iii) $\frac{\partial M}{\partial x} + x = \frac{\partial N}{\partial y} + y$ (iv) $\frac{\partial M}{\partial x} + y = \frac{\partial N}{\partial y} + x$
c)	Which of the following is used to solve second order differential equations? (i) Leibnitz's equation method (ii) Variation of parameter method (ii) Bemoulli's equation method (iii) Variable separable method
d)	The Laplace transform of t^6 is (i) $\frac{6!}{s^7}$ (ii) $\frac{6!}{s^6}$ (iii) $\frac{7!}{s^6}$ (iv) 0
e)	If $\boldsymbol{R} = x\boldsymbol{I} + y\boldsymbol{J} + z\boldsymbol{K}$, then $\boldsymbol{\nabla}.\boldsymbol{R}$ equals to (i) 0 (ii) 1 (iii) 2 (iv) 3
4.	**True or False**
a)	$sinx$ is an example of even function.
b)	Bernoulli's equation can be reducible to Leibnitz's linear equation.

c)	The complete solution of a differential equation is the sum of complementary function and the particular integrals.
d)	Laplace transforms are used to solve linear differential equations with constant coefficients.
e)	Any motion in which the curl of the velocity vector is zero is said to be irrotational.
SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Determine the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.
6.	Solve the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$.
7.	Find the Laplace transform of (i) $t^2 sinat$ (ii) $te^{-t} sin3t$.
8.	Find the work done in moving a particle in the force field $\boldsymbol{F} = 3x^2\boldsymbol{I} + (2xz - y)\boldsymbol{J} + z\boldsymbol{K}$ along (i) the straight line from $(0, 0, 0)$ to $(2, 1, 3)$. (ii) the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Obtain the Fourier series to represent x^2 in the interval $(-l, l)$.
10.	Using the concept of variation of parameter, solve the differential equation $y'' - 2y' + y = e^x logx$.
11.	Determine the inverse Laplace transform of $\frac{4s+5}{(s-1)^2(s+2)}$.
12.	Apply Green's theorem, to evaluate $\int_C [(y - sinx)dx + cosx\, dy]$, where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	(a) Rearrange the Bernoulli's differential equation $x\frac{dy}{dx} + y = x^3y^6$ as a Leibnitz's differential equation and hence solve it. (b) Solve $(D^2 - 1)y = xsin3x + cosx$. (10+10)
14.	Verify Gauss divergence theorem for $\boldsymbol{F} = (x^2 - yz)\boldsymbol{I} - (y^2 - zx)\boldsymbol{J} + (z^2 - xy)\boldsymbol{K}$, taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b, 0 \le z \le c$.
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	Obtain the Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, 0 < x < \pi \end{cases}$. Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.
16.	Apply Laplace transform to solve the differential equation $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$ given that $y(0) = 1,\ y'(0) = 0,\ y''(0) = -2$.

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